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Buckling of Functionally Graded Plates with Varying In-Plane Loading

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ABSTRACT

Functionally graded materials are materials with a spatial variation of material properties. The FGM plates have significant applications in turbine blades, helicopter blades, compressor blades, aircraft or marine propellers. Many of these plates are subjected to in-plane load due to fluid or air pressure. Hence it is necessary to study their behavior under different types of loads .Study of buckling of functionally graded material (FGM) plates with different boundary conditions under varying in-plane load is therefore an important study. In these days, FGM have many engineering applications because of their high stiffness and strength.The analysis is completed utilizing ANSYS programming. In ANSYS, the SHELL 281 component with six degrees of freedom per node is utilized. Twelve by twelve mesh and twelve layers were chosen for the analysis as per the results obtained in convergence study. Buckling of FGM plates with different in plane loading are studied. The effect of different parameters like width to thickness proportion, aspect ratio, gradient index and boundary conditions on the buckling load of FGM plates with varying in-plane load were studied.

INTRODUCTION

Functionally graded materials are materials with a spatial variation of material properties. The volume fractions of two or more materials may be varied continuously either only along the thickness direction or as a function of the in-plane dimensions. FGMs are usually made from a mixture of metals and ceramic. In this thesis, we consider plates in which the material properties change continuously through the thickness.

These FGM plates are used in various engineering applications and they are often subjected to dynamic loading. FGMs are used by the engineering community mainly in nuclear plants and space craft of high temperature applications as FGM scan with stand high temperatures. Presently they are also used in structural walls, body coatings for cars, and in sports products. In the FGMs, interface problems are eliminated by changing the volume fraction of constituent materials smoothly and continuously from surface to surface.

Many studies have been conducted on free vibration analysis and dynamic response of the functionally graded material plates subject to dynamic loading. Functionally graded materials, due to their mechanical and thermal merits compared to singly composed materials, have been widely used for a variety of engineering applications. FGMs have different applications especially for aerospace, automobiles, industries and engineering structures and electronics.

LITERATURE REVIEW

Plenty of studies for vibration, thermal stress and thermal bending of the functionally graded plates are available in the literature.

Praveen and Reddy(1998) investigated then on linear static and dynamic response of functionally graded ceramic–metal plates in a steady temperature field. The plate was subjected to dynamic transverse loads. They used the finite element method (FEM) based on the first-order shear deformation plate theory (FSDPT).

Reddy(2000) investigated functionally graded plates, based on the third-order shear deformation plate theory. Numerical results of the linear third-order theory and non-linear first-order theory were presented to show the effect of the material distribution on the deflections and stresses.

A semi-analytical solution for the nonlinear vibration of laminated FGM plates with geometric imperfections was presented by Yang *et al.* (2001). They showed that the vibration frequencies are very much dependent on the vibration amplitude and the imperfection mode.

Chung and Chi (2001) studied functionally graded material (FGM) plate of medium thickness subjected to transverse stacking.

Javaheri and Eslami (2002) investigated rectangular functionally graded plates (FGPs) using the variational methodology. The vibration characteristics and transient response of shear-deformable functionally graded plates and panels in thermal environments was studied by Yang and Shen (2002, 2003). They considered material properties to be temperature-dependent and the effect of temperature rise on the dynamic response was reported.

Lanhe(2004)studied stability of a rectangular plate made of functionally graded material (FGM) under loads.

Kang and Leissa (2005) formulated an exact solution for the buckling analysis of rectangular plates having two opposite edges simply supported. These edges were subjected to linearly varying normal stresses.

Abrate (2006) presented free vibration, buckling, and static deflections of functionally graded plates. Heshowed that natural frequencies of the functionally graded plates were proportional to the homogeneous isotropic plates.

Ebrahimi and Rastgo (2008) investigated the free vibration of smart FGM plates. They studied FGM plates integrated with two uniformly distributed actuator layers made of piezoelectric (PZT4) material on the top and bottom surfaces of the circular FG plate based on the classical plate theory (CPT).

Alinia *et al.* (2012) analysed inelastic clasping and post clasping conduct of stocky plates under joined shear and in-plane twisting hassles. They compared the results with slim plates.

Wattanasa kulpong *et al.* (2012) investigated the free vibration of functionally graded beams with general elastic end constraints by DTM. The boundary conditions were arbitrary, and various types of elastically end constraints were analyzed. Hadi *et al.* (2013) presented an elastic analysis of transverse loading acting on the functionally graded beam. They determined the stresses and strains on FG beam by using energy method, and also used power law for varying thickness. This gave the exact solution for stresses and displacements.

Objective of Present Study

FGM has many applications in engineering and aerospace structures. The idea arose out of making a composite material by varying the micro structure from one material to another material with a specific gradient. Composite materials have greater advantages than the materials they are composed of. The plates are often subjected to in-plane forces due to components of aerodynamic or hydrodynamic forces acting on it. There are many studies on the free vibration of FGM plates, but vibration and buckling of FGM plates subjected to loading such as in turbines have not been studied much. Functionally graded materials have good thermal resistance and high stability in thermal conditions. Hence, the present work is to determine the buckling behavior of FGM plates subjected to in-plane varying loads.

Outline of The Present Work

The present study deals with functionally graded material plates subjected to in plane loading and their behavior under the influence of different parameters like aspect ratio, side width ratio and gradient index and also different boundary conditions.

This thesis contains five chapters. A brief review is presented here.

In chapter 1, an introduction of functionally graded plates and applications of FGM plates is presented.

In chapter 2, the literature review pertaining to the previous work done in this area is detailed. An objective of the present study is also outlined in this chapter.

In chapter 3, a brief theory and formulation of finite element method is given. Methodology of buckling of FGM plates in ANSYS is explained in this chapter.

In chapter 4, detailed discussion about results of buckling of FGM plates by using ANSYS software is given. The plates are subjected to different in-plane loading and the effect of the different parameters like aspect ratio, side width ratio and gradient index on buckling load is presented.

In chapter 5, conclusions drawn from the studies are outlined.

THEORY AND FORMULATION

Governing Differential Equation

A twisted FGM plate is shown in figure3.1.

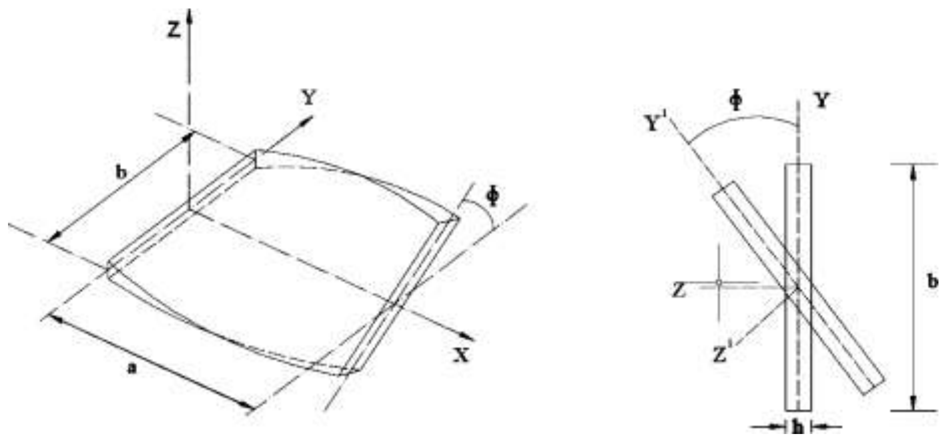

Figure 3.1: FGM twisted plate

In the figure2.1,the dimensions„ a‟and„b‟ denote the length and width of the plate respectively and Φ is the angle of twist which in this case is zero as a flat plate is considered. „h‟ is the thickness of the plate.
Figure2.2 shows a differential element of the twisted panel. N_x,and$N_{xy}$are the internal axial forces,$Q_x$and $Q_y$are the shear forces andM_x,$M_y and\ M_{xy}$are the moment resultants.

Constitutive Relations

The FGM plate taken for the study is made up of one side metal and the other ceramic. A parameter „n‟ (material property index) shows the material variation along the thickness. The plate is fully ceramic for n = 0, and the plate is fully metal for n = α. Material properties are dependent on the n value and the position in the plate and vary according to the power law, (Reddy[11]), i.e.,

$$P_z=(P_t-P_b)V_f+P_b \qquad (3.3)$$

where P is the relevant material property, $P_t$and $P_b$are the material property at the top and bottom surfaces respectively and z is the location along the thickness measured from the mid- surface.$V_f$is the volume fraction index and n is the material property index. Young‟s modulus

(E) varies according to the power law. Material density (ρ) and Poisson‟s ratio (μ) are assumed to be constant.
The material property of the FGM changes through the thickness. The numerical model is broken up into a number of layers in order to model the gradual change in properties of the FGM. Each layer is assumed to be isotropic. Material properties are calculated at the mid-point of each of these layers from the mid surface using power law, (Reddy [11]). Although the layered structure does not reflect the gradual change in material property, by using a sufficient number of layers the gradation can be approximated.

Finite element formulation

Finite element software ANSYS 13.0 is used for the formulation of the FGM plate. SHELL 281 is used to model the

element. SHELL 281 has eight nodes with six degrees of freedom per node: three translations in x-, y- and z- directions and three rotations. The first-order shear deformation theory is assumed for the modeling in ANSYS 13.0.
The plate is assumed to be made up of layers, where each layer is considered to be homogeneous and isotropic.

Strain Displacement Relations

Green-Lagrange"s strain displacement relations are used. The elastic stiffness matrix is derived from the linear part of the strain and the geometric stiffness matrix from the nonlinear strain components.

ANSYS Methodology

Problems that can be solved in ANSYS include static/dynamic structure analysis (both linear and non-linear), heat transfer and fluid problems, as well as acoustic and electro-magnetic problems. There are three stages:

Preprocessing

This is to define the problem. The steps are

1. Define key points/lines/areas/volumes
2. Define element type and material/geometric properties
3. Meshing of lines or areas or volumes

Solution

Here loads and constraints are assigned and problem is solved.

Post-processing

This stage is to view and process the results like the list of nodal displacements, element forces and moments and deflection plots.

The present problem has been solved using ANSYS software. The FGM plate was first solved without loading in order to validate the methodology and the results compared to previous results for free vibration and buckling. Also the methodology was tested for a laminated composite plate with different types of edge loading and results compared to a result from a previous paper.

The results matched closely in most cases. Then the software was run for studying the FGM plate with different types of in-plane loads.

RESULTS AND DISCUSSIONS

Introduction

The FGM plate considered here consists of ceramic on top and metal at the bottom. In FGM plates, the material properties change continuously over the thickness by varying the gradient index (n). Those properties are density (ρ), Young"s modulus (E) and also Poisson"s (υ) ratio. If n = 0, then the plate is completely ceramic and if n = infinity then the plate is completely metal. Material properties depend on gradient index (n). Byusing the power law (Reddy[2000]) we can find out the material properties.

$$P(z)=(P_t-P_b)V+P_b \qquad (4.1)$$

$$V_f=(z/h+½)^n \qquad (4.2)$$

Here P represents a material property like Young"s modulus.P_t and $P_b$represents the relevant property at the top and bottom faces of the plate, h denotes the total thickness of the plate, n represents the gradient index or the material variation profile through thickness, $V_f$is the volume fraction and z is the thickness varyiation from the mid plane.

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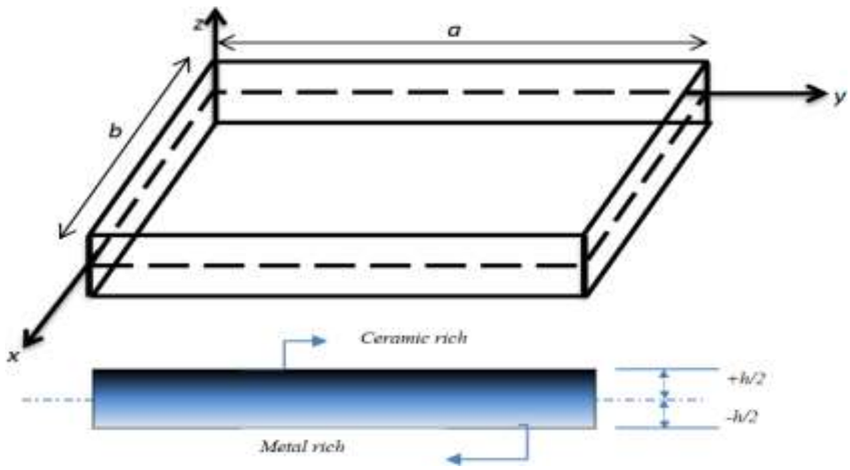

Figure 4.1: FGMplate

By using the MATLAB software, the Young‟s modulus of each layer was calculated. Poisson‟s ratio and density were kept constant. Then a model of FGM plate was developed by using ANSYS. Free vibration of cantilever FGM flat plate will be studied and results obtained and these results compared with previous results. Then the flat FGM plate is modeled and again results will be studied for plate with varying in plane loads.

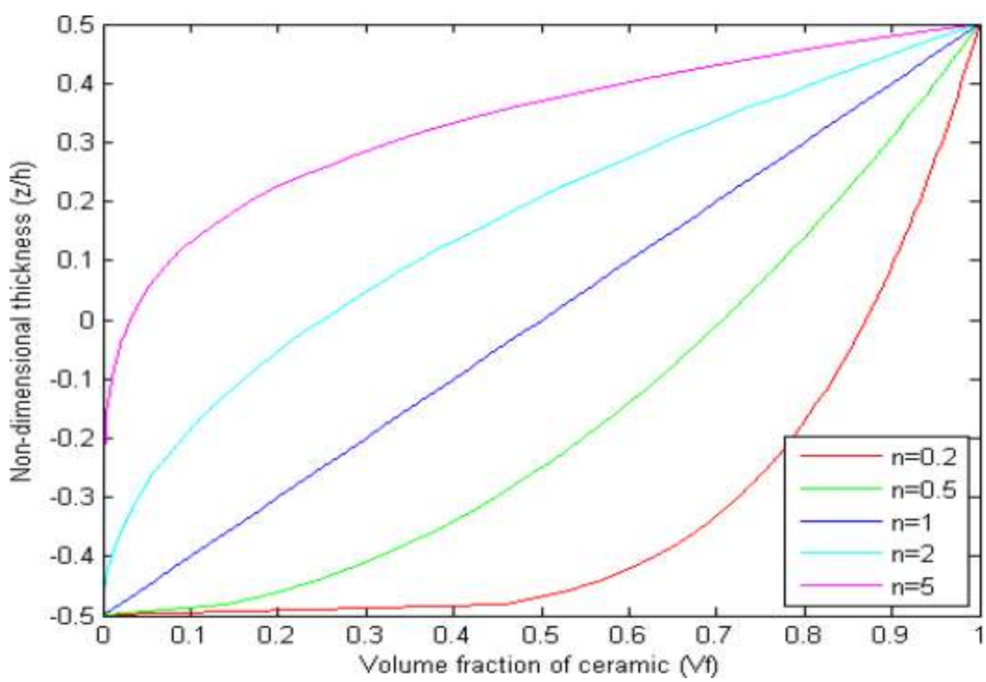

Figure 4.1 Shows The Variation Of V_f Through The Plate Thickness.

Figure4.2: Variation of Volume fraction (V_f) through plate thickness
For n=1 the material properties changes linearly, n = 0 plate is total titanium and n= infinite plates is purely aluminum oxide

RESULTS AND DISCUSSIONS

The results of the free vibration analysis of functionally graded material cantilever plate (CFFF), simply supported plate (SSSS), and all edges clamped (CCCC) are presented. In ANSYS, an eight noded shell element SHELL281isused to model the FGM plate. This element has six degrees of freedom per node. As the material property of the FGM changes through the thickness, the numerical model is broken up into various layers in order to model the change in properties. Each layer is assumed to be isotropic. Material properties are calculated at the mid- point of each layer from the mid surface using power law. Although the layered structure does not reflect the actual gradual change in material property, by using a sufficient number of layers the gradation can be approximated. A convergence study was done to decide on the number of layers required to model accurately the FGM plate and mesh size for greater accuracy of results.

Convergence Study

For this study FGM plate consisting of aluminum oxide in a titanium (Al/Al_2O_3) matrix and square plate with aspect ratio a/b=1, b/h=100, (where a, b, and h are the width, length, and thickness) was taken. The properties of constituents are E_c=380GPa, ϑ = 0.3, for aluminum oxide and E_m= 70GPa, ϑ = 0.3, for Titanium.
The convergence study on simply supported flat FGM plate with gradient index n = 0 for different mesh divisions is shown in Table 4.1.The results show better convergence for 12×12 mesh division. The 12×12 mesh division is used for the further study.

Table 4.1: Convergence of non-dimensional buckling load (λ)of simply supported flat FGM plate (n = 0) with varying mesh size a/b=1, b/h=100,n =0

Mesh size	Buckling load (N_o) KN	Non-dimensional Buckling load()
4 ×4	696.59	19.90
66	687.00	19.63
8 ×8	686.60	19.62
10 ×10	686.53	19.61
12 ×12	686.52	19.61
	Reddyetal.[14]	19.57

The convergence study is done by using simply supported flat FGM plate with varying number of layers using gradient index n =1. The observations are given in Table 4.2. From the observations, it is concluded that 12 numbers of layers are sufficient to represent FGM property as an equivalent laminate section.

Table 4.2: Convergence of non-dimensional buckling load(λ)of simply supported flat FGM plate (n=1) with varying number of layers a/b =1, b/h =100, n =1

Noof layers	Buckling load (N_o) KN	Non-dimensional Buckling load()
4	352.87	10.08
6	349.09	9.97
8	347.74	9.93
10	347.10	9.91
12	346.80	9.91
	Reddyetal.[14]	9.77

Buckling of FGM plate within-plane loading
To validate the ANSYS formulation for in-plane loading, buckling of simply supported laminated composite plates within-plane loading was first done. The results were compared with previous studies by Zhong and Gu [18]. The loading is given by the expression for compressive force

$$N_x=N_0(1-\eta \frac{y}{b}) \quad (3.3)$$

Table4.3: Comparison of non-dimensional buckling load factors of symmetric cross-ply square plate withη=0,0.5, 1

	η=0	η=0.5	η=1
Present study	21.698	28.70	40.268
Zhong andGu [18]	22.317	29.432	40.999

The above table shows that the results are quite comparable with that of Zhong and Gu [18].

Numerical Results:-

As the formulation was validated in ANSYS, the effect of various parameters on the buckling of flat FGM plates with varying in-plane load was studied. Based on the convergence study, a mesh size of 12×12 and 12 number of layers was taken throughout the study. Table 4.4 shows the non-dimensional buckling load of flat FGM plates with different boundary conditions subject to linearly varying load by using different parameters like b/h ratio, aspect ratio(a/b) and gradient index(n).It is observed from Table 4.4 that for the simply supported plate, as the b/h ratio increases, the buckling load increases till about b/h= 200.

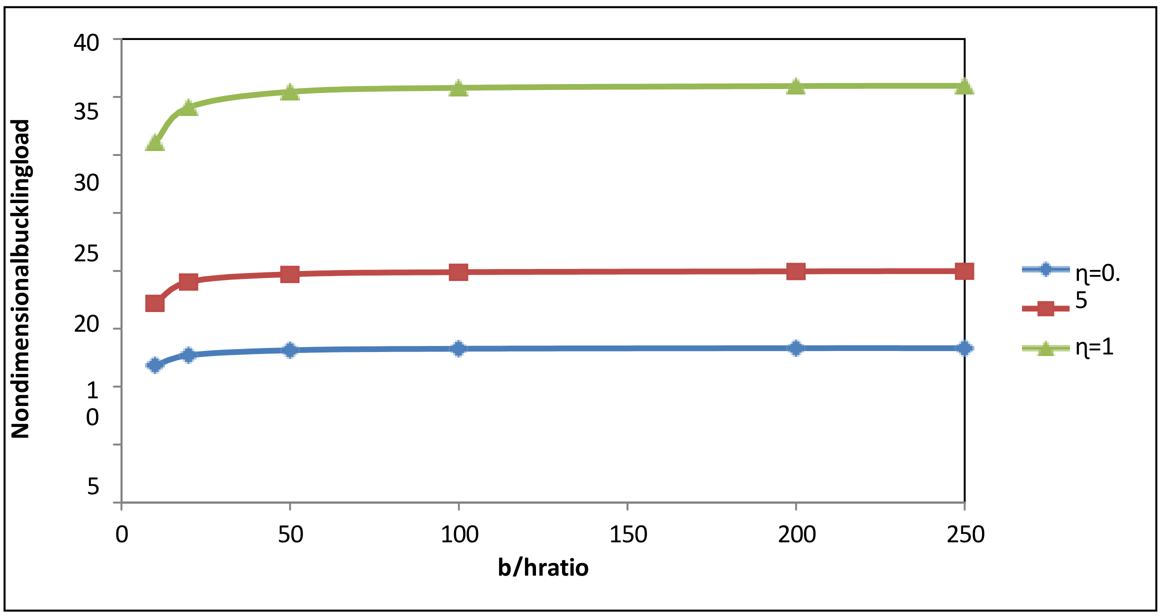

Figure 4.4: **Variation of non-dimensional buckling load with b/h ratio and load parameter (η)for SSSS plate and a/b =1**

Variation of the buckling load with aspect ratio for a simply supported plate with various loading conditions is shown in Table 4.5. The buckling load is found to decrease as the aspect ratio increases.

In Table 4.6, a plate with all edges simply supported is analysed for varying gradient index. It is observed here that as the gradient index increases, the buckling load decreases for all types of varying load.

CONCLUSIONS

The behavior of buckling of functionally graded material (FGM) plate subjected to various types of in-plane loading was studied. The work has been done in ANSYS. The effects of geometrical parameters like side width ratio, aspect ratio (a/b), gradient index and different boundary conditions on buckling parameters of FGM plates has been studied.

From the above study, it is observed that for all boundary conditions, with increasing side width ratio, non-dimensional buckling load increases.

Also for simply supported and cantilever plates, while increasing the aspect ratio and gradient index, the non-dimensional buckling load decreases. In fact for cantilever plates, there seems to be no effect of increase in gradient index for all loading types beyond n =10.

But in plate with all edges clamped, non-dimensional buckling load decreases up to a/b=1 and with further increase in aspect ratio, the buckling load parameter increases.

The loading with η = 1.5 gives highest buckling load value as compared to the trapezoidal loading (η = 0.5) and uniformly varying load (η = 1).

Based on above conclusions, the FGM plate maybe suitably designed for different loading types so that buckling load may be optimized.

Scope for the Future Work

In this paper, the behavior of buckling of FGM plates with in-plane loading with different boundary conditions was studied. In this study, ceramic and metal were assumed as FGM material. The study may be extended to different material of FGM and their buckling behavior while applying in-plane loading with different boundary conditions may be studied.

The thermal buckling of FGM plates with different boundary conditions subjected to in-plane loading and the effect of different parameters like aspect ratio, side depth ratio and gradient index may also be investigated further.

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