EDUZONE: International Peer Reviewed/Refereed Multidisciplinary Journal (EIPRMJ), ISSN: 2319-5045
Volume 12, Issue 2, July-December, 2023, Available online at: www.eduzonejournal.com

Innovative Approaches to Address Multi-Objective Fractional Programming Problems using Advanced Mean Deviation Techniques and Point Slopes Formula

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Introduction

This dissertation explores a cutting-edge research problem at the interface between mathematics and optimization. This study investigates brand-new approaches to complicated multi-objective fractional programming problems. In order to give effective solutions for issues with numerous competing objectives and fractional limitations, it makes use of sophisticated mean deviation techniques and the point slopes formula.

Literature Review and Problem Statement

The part of this article illustrates and cites some relevant works and theories in relation to this direction. According to the author Mustafa, et al. 2021, numerous fields, including business, factory planning, financial and corporate companies, health care and infirmary planning, etc., have used the "fractional programming problem (FPP)", which has been used as a key designing agency. FPP comes up frequently in applications of decision-making, such as game theory, transportation, etc. The term "multi-objective programming" refers to an optimum issue in which two or more goals must be optimized, reduced, or both maximized while taking into account certain limitations (Mustafa, *et al.* 2021). If switching to a different solution does not accomplish all the goals, the original solution is effective. The feasibility zone is likewise impacted by the equivalence, and numerical examples are provided to help the concept. According to the author Yesmin, 2021, an important kind of optimization problem known as a multi-objective optimization problem (MOOP) allows users to simulate a wide range of real-world applications. In the process of developing corporate and economic planning, a study of quadratic programming is offered. To resolve MOOP, an advanced transformation mechanism has been suggested. Many mathematical operations have been developed specifically for use in OR applications. These methods, which are based on fundamental mathematical ideas, have developed into significant areas of specialization for business operations(Yesmin, 2021). The Pareto optimum solution and M-Pareto optimal solution for convex MOQPP were structurally analyzed in this study.

The complexity of optimizing systems with numerous objectives and fractional limitations is the issue that this topic attempts to solve. Such situations are difficult for traditional optimization techniques to manage, which results in less-than-ideal choices. This study aims to address this problem by presenting novel methods based on the formulas for mean deviation and point slopes (Mustafa, *et al.* 2022).

Some Related Definitions

Canonical Form of Linear Programming (LP)

A standardized model which simplifies the procedure of calculating the solution of linear programming problems is known as the canonical form of a linear programming (LP) problem. An LP problem is generallydenoted as follows in canonical form,

Maximize (or minimize) $Z = c_1x_1 + c_2x_2 + ... + c_nx_n$

Subject to:

$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \leq b_1$

$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n \leq b_2$S

...

$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n \leq b_m$

Where:

- Z is the objective function to be maximized or minimized.
- $c_1, c_2, ..., c_n$ are coefficients expressing the contribution of each variable ($x_1, x_2, ..., x_n$) to the objective function
- a_{ij} means the coefficients of the constraints.

- $b_1, b_2, ..., b_m$ are the right-hand sides of the constraints.
- $x_1, x_2, ..., x_n$ are the decision variables.

Vertex

A vertex (or corner point) is a key idea related to a problem's feasible area in the context of linear programming. The set of all conceivable combinations of choice variables that satisfies the system of linear constraints is the feasible zone (Yan, *et al.* 2019).

Feasible Solution

A point in the decision variable space that meets every constraint of the issue is referred to as a "feasible solution" in "linear programming". It is a solution that respects the linear constraints and is thus inside the feasible zone, which is determined by the set of values for the decision variables.
A pair of specific values for (x1,x2) is said to be a feasible solution if it satisfies all the constraints. (x1,x2) = (0,0) and (x1,x2) = (1,1) are feasible.
(x1,x2) = (1,−1) and (x1,x2) = (1,2) are not feasible.

Differentiable

Differentiability in the context of mathematical functions refers to a feature of a function that enables it to have a derivative at a certain point. Differentiability in linear programming frequently has to do with the constraints and objective function. A differentiable function is a function that can be approximated locally by a linear function. [f(c + h) − f(c) h] = f (c). The domain of f is the set of points c ∈ (a, b) for which this limit exists. If the limit exists for every c ∈ (a, b) then we say that f is differentiable on (a, b).

The point-slopes formula is used to examine how changes in resource availability affect the best solution. It is derived from the shadow price (dual value) of restrictions (Sudhoff, 2021).
The point-slope formula is represented below.
Y–y1 =m(x–x_1), Where, m is the slope of the line. x_1 is the coordinates of the x-axis.

Mean Deviation (MD)

The average absolute difference between data points and their mean is measured by the mean deviation, often known as the mean absolute deviation (MAD). Mean deviation can be used in linear programming as an objective function or constraint to reflect circumstances in which minimizing variability or departure from a target value is crucial.
These definitions offer a thorough knowledge of the fundamental ideas in linear programming, optimization, and other related mathematical techniques used to simulate and resolve real-world problem-solving situations.
The formula to calculate the mean deviation for the given data set is given below.
Mean Deviation = [Σ |X – μ|]/N. ……………………………(4)
Here, Σ represents the addition of values. X represents each value in the data set.

Multi-Objective Fractional Programming Problem

Multi-Objective Linear Programming Problem (MOLFPP)

A mathematical optimization problem known as a multi-objective linear programming problem (MOLFPP) requires the simultaneous optimization of several competing linear objectives. The following is a definition of it:
Maximize (or Minimize) $Z_1 = c_{11}x_1 + c_{12}x_2 + ... + c_{1n}x_n$
Maximize (or Minimize) $Z_2 = c_{21}x_1 + c_{22}x_2 + ... + c_{2n}x_n$
...
Maximize (or Minimize) $Z_k = c_{k1}x_1 + c_{k2}x_2 + ... + c_{kn}x_n$

Subject to
$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \leq b_1$
$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n \leq b_2$
...
$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n \leq b_m$
Where:
- $Z_1, Z_2, ..., Z_k$ represent the k linear objectives to be maximized or minimized.
- c_{ij} are coefficients of the linear objectives.
- $x_1, x_2, ..., x_n$ are the decision variables.

- a_{ij} are coefficients of the constraints.
- $b_1, b_2, ..., b_m$ are the right-hand sides of the constraints.

Multi-Objective Quadratic Fractional Programming Problem (MOQFPP)

An optimization issue that expands the multi-objective framework to incorporate quadratic and fractional objectives is known as a Multi-Objective Quadratic Fractional Programming Problem (MOQFPP). it may be stated Mathematically conveying as follows:

Maximize (or Minimize) $Z_1 = (c_{11}x_1 + c_{12}x_2 + ... + c_{1n}x_n) / (d_{11}x_1 + d_{12}x_2 + ... + d_{1n}x_n)$
Maximize (or Minimize) $Z_2 = (c_{21}x_1 + c_{22}x_2 + ... + c_{2n}x_n) / (d_{21}x_1 + d_{22}x_2 + ... + d_{2n}x_n)$
...
Maximize (or Minimize) $Z_k = (c_{k1}x_1 + c_{k2}x_2 + ... + c_{kn}x_n) / (d_{k1}x_1 + d_{k2}x_2 + ... + d_{kn}x_n)$

Subject to
$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \leq b_1$
$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n \leq b_2$
...
$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n \leq b_m$
Where:
- $Z_1, Z_2, ..., Z_k$ represent the k objectives with both quadratic and fractional components.
- c_{ij} and d_{ij} are coefficients of the linear and denominator terms of the objectives.
- $x_1, x_2, ..., x_n$ are the decision variables.
- a_{ij} are coefficients of the constraints.
- $b_1, b_2, ..., b_m$ are the right-hand sides of the constraints.

MOQFPPs appear when the objectives entail both nonlinear (quadratic) and fractional connections, necessitating the use of a particular technique to locate the best solutions that balance these intricate objective functions.

MOFPP Solution Techniques

Finding solutions that maximize many competing objectives with fractional components is the goal of solving "Multi-Objective Fractional Programming Problems (MOFPPs)".

The Derivation of the Study

The study of MOFPPs is inspired by real-world situations where decision-makers must make trade-offs between a variety of non-linear objectives. Due to the possibility of both fractional and non-linear components in these objectives, typical linear programming is insufficient (Gonzales-Zurita, *et al.* 2020). The Pareto frontier is a range of possible solutions that decision-makers can choose from in order to best meet their preferences and limitations.

Algorithm

The "Weighted Sum Method" is one strategy that is frequently used to solve MOFPPs. By giving each objective a weight, it reduces the multi-objective issue to a single-objective problem with a single composite objective.
This algorithm enables decision-makers to weigh several competing objectives in MOFPPs while still making well-informed decisions.

Numerical Example

Here it is considered a numerical example. Suppose we have a portfolio optimization problem where we aim to allocate investments of three assets (A, B, and C) to maximize returns while minimizing risk and we want to explore different mean deviation techniques.

Using these methods, we may come up with multiple portfolio allocations that maximize return and mean deviation while accepting differing risk appetites and taking asset correlations into account (Binder, *et al.* 2020). Each approach offers a different viewpoint on portfolio optimization, enabling investors to make well-informed choices that are suited to their individual objectives and risk tolerance.

Discussion and Comparison

Insightful information on how investors may make well-informed decisions while balancing risk and return is provided by the comparison and explanation of the various portfolio optimization mean deviation methods. The first method is the simplest, calculating mean deviation as the average of absolute deviations from the portfolio's return(Gupta, *et al.* 2020). The next technique incorporates asset correlations to give a more precise representation of portfolio risk. Then, by taking risk aversion into account, Chandra Sen's technique enables investors to express their choices more succinctly(Savage, *et al.*2021). Then Advanced Harmonic Average Technique combines mean and standard deviation to produce a harmonic mean deviation.

Conclusion

The importance of managing risk and return while making investment decisions is highlighted by the examination of portfolio optimization techniques employing mean deviation methodologies. Investors can use a variety of tools provided by these techniques to tailor their portfolios to their particular goals and risk appetite. Ultimately, these mean deviation tactics help investors navigate the complex financial landscape, come to informed conclusions, and create portfolios that are customized to their unique needs.

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