Study of Exponential Function in Algebraic Expressions

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ABSTRACT

The exponential function as a numerical concept assumes a significant part in the corpus of numerical information, however tragically understudies have issues getting a handle on it. This paper concentrates on the instances of exponential function application in a certifiable setting. The area of any exponential function is the set of every genuine number. An exponential function has no upward asymptote. Each exponential function has one level asymptote. The chart of any exponential function is either expanding or diminishing.

Keywords: mathematical modeling, exponential function, real, functional knowledge.

INTRODUCTION

Exponential functions are hard to grasp by students. Furthermore, this kind of function is the foundation of math and differential conditions and bunches of normal and social peculiarities could be made sense of through the models in view of exponential functions. At last, the incredible significance of information about exponential functions can be made sense of as "The best inadequacy of humanity is our powerlessness to figure out the exponential function". Barlett additionally contends that despite the fact that the development is the underpinning of our civilization success in the feeling of business, economy or innovation, this point isn't sufficiently addressed in the numerical and physical science training. The cycles of development are overwhelming in our regular daily existence, and science of development is arithmetic of the exponential function. It is critical to comprehend the exponential development since that would make us ready to assess numerous circumstances concerning development [1].

EXPONENTIAL FUNCTION

The exponential function is a kind of numerical function which are useful in tracking down the development or rot of populace, cash, cost, and so forth that are developing or rot exponentially. Jonathan was perusing a news story on the most recent exploration made on bacterial development. He read that an investigation was led with one bacterium. After the main hour, the bacterium multiplied itself and was two in number. After the subsequent hour, the number was four. At each hour the quantity of microscopic organisms was expanding. He figuring would be the quantity of microscopic organisms following 100 hours assuming that this example proceeds. At the point when he got some information about a similar the response he got was the concept of an exponential function [2].

What is Exponential Function?

Exponential function, as its name proposes, includes examples. Yet, note that, an exponential function has a steady as its base and a variable as its type yet not the alternate way round (in the event that a function has a variable however the base and a consistent as the example then it seems to be a power function yet not an exponential function). An exponential function can be in one of the accompanying structures [3].

Exponential Function Definition

In science, an exponential function is a function of structure f (x) = hatchet, where "x" is a variable and "a" will be a steady which is known as the foundation of the function and it ought to be more noteworthy than 0.

An exponential function can be in one of the following forms:

→ $f(x) = b^x$
→ $f(x) = ab^x$
→ $f(x) = ab^{cx}$
→ $f(x) = e^x$
→ $f(x) = e^{kx}$
→ $f(x) = p\,e^{kx}$

Here $b>0$ and $b \neq 1$

Exponential Function Examples

Here are some examples of exponential function [4].

$f(x) = 2^x$

$f(x) = (1/2)^x$

$f(x) = 3e^{2x}$

$f(x) = 4\ (3)^{-0.5x}$

Exponential Function Formula

A basic exponential function, from its definition, is of the form $f(x) = b^x$, where 'b' is a constant and 'x' is a variable. One of the popular exponential functions is $f(x) = e^x$, where 'e' is "Euler's number" and e = 2.718... If we extend the possibilities of different exponential functions, an exponential function may involve a constant as a multiple of the variable in its power. i.e., an exponential function can also be of the form $f(x) = e^{kx}$. Further, it can also be of the form $f(x) = p\ e^{kx}$, where 'p' is a constant [5].

Thus, an exponential function can be in one of the following forms.

$$f(x) = b^x$$

$$f(x) = ab^x$$

$$f(x) = abc^x$$

$$f(x) = e^x$$

$$f(x) = e^{kx}$$

$$f(x) = p\ e^{kx}$$

Here, apart from 'x' all other letters are constants, 'x' is a variable, and f(x) is an exponential function in terms of x. Also, note that the base in each exponential function must be a positive number. i.e., in the above functions, $b > 0$ and $e > 0$. Also, b should not be equal to 1 (if $b = 1$, then the function $f(x) = bx$ becomes $f(x) = 1$ and in this case, the function is linear but NOT exponential).

The exponential function arises whenever a quantity's value increases in exponential growth and decreases in exponential decay. Some more differences between exponential growth and decay along with their formulas in the following table are given [6] :

Exponential Growth	**Exponential Decay**
In exponential growth, a quantity slowly increases in the beginning and then it increases rapidly.	In exponential decay, a quantity decreases very rapidly in the beginning, and then it decreases slowly.
The exponential growth formulas are used to model population growth, to model compound interest, to find doubling time, etc	The exponential decay is helpful to model population decay, to find half-life, etc.
The graph of the function in exponential growth is increasing.	The graph of the function in exponential growth is decreasing.
In exponential growth, the function can be of the form: $f(x) = ab^x$, where $b > 1$. $f(x) = a (1 + r)x$ $P = P0\ e^{k\ t}$ Here, $b = 1 + r \approx e^{k.}$	In exponential decay, the function can be of the form: $f(x) = ab^x$, where $0 < b < 1$. $f(x) = a (1 - r)x$ $P = P0\ e^{-k\ t}$ Here, $b = 1 - r \approx e^{-\ k.}$

In the above formulas,

a (or) P_0 = Initial value
r = Rate of growth
k = constant of proportionality
x (or) t = time (time can be in years, days, (or) months. Whatever we are using should be consistent throughout the problem).

Exponential Function Graph

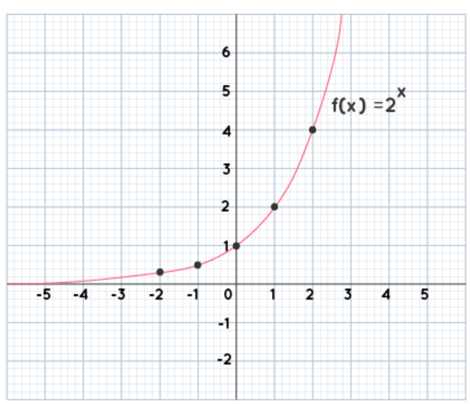
We can understand the process of graphing exponential function by taking some examples. Let us graph two functions $f(x) = 2^x$ and $g(x) = (1/2)^x$. To graph each of these functions, we will construct a table of values with some random values of x, plot the points on the graph, connect them by a curve, and extend the curve on both ends. The process of graphing exponential function can be learned in detail by clicking here [7].

Here is the table of values that are used to graph the exponential function $f(x) = 2^{x.}$

Here is the table of values that are used to graph the exponential function $g(x) = (1/2)^{x.}$

$f(x) = 2^x$

x	f(x)	
-2	0.25	$2^{-2} = \frac{1}{4} = 0.25$
-1	0.5	$2^{-1} = \frac{1}{2} = 0.5$
0	1	$2^0 = 1$
1	2	$2^1 = 2$
2	4	$2^2 = 4$



Note: From the above two graphs, we can see that $f(x) = 2^x$ is increasing whereas $g(x) = (1/2)^x$ is decreasing. Thus, the graph of exponential function $f(x) = b^x$.
increases when $b > 1$
decreases when $0 < b < 1$

Exponential Function Asymptotes

The exponential function has no upward asymptote as the function is consistently expanding/diminishing. However, it has a flat asymptote. The condition of level asymptote of an exponential function f(x) = abx + c is generally y = c. i.e., it is only "y = consistent being added to the example part of the function". In the over two charts (of f(x) = 2x and g(x) = (1/2)x), we can see that the level asymptote is y = 0 as nothing is being added to the type part in both the functions. Accordingly,

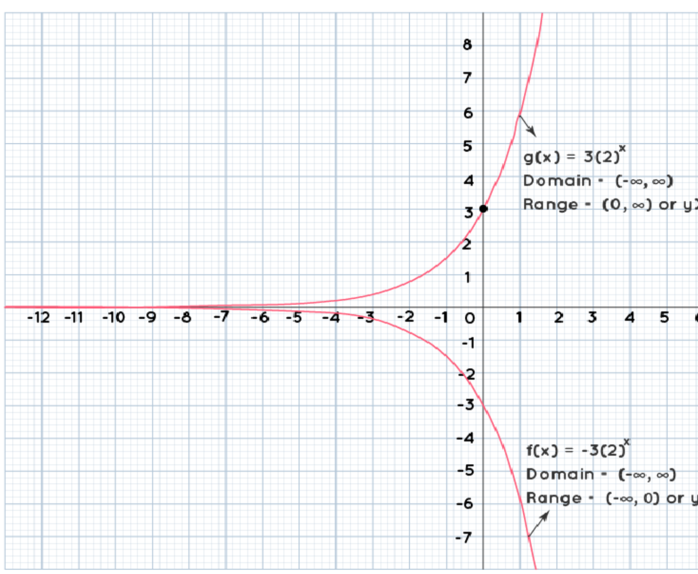
An exponential function never has an upward asymptote.

The level asymptote of an exponential function f(x) = abx + c is y = c.

Space and Range of Exponential Function

We know that the space of a function y = f(x) is the set of every x-esteem (inputs) where it tends to be processed and the reach is the set of all y-values (yields) of the function. From the diagrams of f(x) = 2x and g(x) = (1/2)x in the past segment, we can see that an exponential function can be registered at all upsides of x. Hence, the space of an exponential function is the set of every genuine number (or) $(-\infty, \infty)$. The scope of an exponential function not entirely set in stone by the even asymptote of the chart, say, y = d, and by seeing whether the chart is above y = d or underneath y = d. Consequently, for an exponential function f(x) = abx,

Space is the set of every genuine number (or) $(-\infty, \infty)$.
Range is $f(x) > d$ if $a > 0$ and $f(x) < d$ if $a < 0$.
To comprehend this, you can see the model underneath.



Exponential Series

The real exponential function can be commonly defined by the following power series,
$e_x = \Sigma n = 0^{\infty} \quad x^n/n! = 1 + (x/1) + (x^2/2) + (x^3/6) + \ldots$

Expansion of some other exponential functions are given as shown below,
$e = \Sigma n = 0^{\infty} \; 1^n/n! = 1 + (1/1) + (1/2) + (1/6) + \ldots$
$e^{-1} = \Sigma n = 0^{\infty} \; (-1)^n/n! = 1 - (1/1) + (1/2) - (1/6) + \ldots$

Exponential Function Rules

The rules of exponential function are as same as that of rules of exponents. Here are some rules of exponents.

Law of Zero Exponent: $a^0 = 1$

Law of Product: $a^m \times a^n = a^{m+n}$

Law of Quotient: $a^m/a^n = a^{m-n}$

Law of Power of a Power: $(a^m)^n = a^{mn}$

Law of Power of a Product: $(ab)^m = a^m b^m$

Law of Power of a Quotient: $(a/b)^m = a^m/b^m$

Law of Negative Exponent: $a^{-m} = 1/a^m$

Apart from these, we sometimes need to use the conversion formula of logarithmic form to exponential form which is [8]:

$b^x = a \Leftrightarrow \log b^{\,a} = x$

Equality Property of Exponential Function

According to the equality property of exponential function, if two exponential functions of the same bases are the same, then their exponents are also the same. i.e.,

$b^{x1} = b^{x2} \Leftrightarrow x^1 = x^2$

Exponential Function Derivative

Here are the formulas from differentiation that are used to find the derivative of exponential function.

$d/dx\ (e^x) = e^x$
$d/dx\ (a^x) = a^x \cdot \ln a.$

Integration of Exponential Function

Here are the formulas from integration that are used to find the integral of exponential function.

$\int ex\ d^x = e^x + C$
$\int ax\ d^x = a^x\ /\ (\ln\ a) + C$

CONCLUSION

Described models start with a complicated genuine circumstance where in the first place it was not clear which sort of numerical techniques ought to be executed. The utilization of innovation permits to investigate, picture and compute prompts the numerical arrangements of this present reality issue. One of the difficulties of contemporary numerical instruction is to make the information on understudies pertinent and appropriate for the requests of the contemporary society. While noticing the presented issue, the introduced concentrate on spans the instructive hypothetical examination and practice. In the field of informative viewpoints of numerical demonstrating, it addressed the significant inquiries concerning study hall guidelines happening in the homeroom, which is thought of as one of vital issue of the numerical displaying processes application in schooling. The numerical substance, for example, the exponential function has an extraordinary significance in true application since the association with various fields of daily existence and sciences is clear, like science, science, innovation, physical science, measurements, designing, telecom, climate, economy, etc. Dominating such a point during secondary school training ought to be perceived as an interest of contemporary society.

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